Sequences and Series

Question1

If $S_n = 1^3 + 2^3 + \ldots + n^3$ and $T_n = 1 + 2 + \ldots + n$, then

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Options:

A.

$S_n = T_{n^3}$

B.

$S_n = T_n^3$

C.

$S_n = T_{n^2}$

D.

$S_n = T_n^2$

Answer: D

Solution:

As we know,

$S_n = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

and $T_n = 1 + 2 + 3 + \ldots + n$

$= \frac{n(n+1)}{2}$

Hence, $S_n = T_n^2$

Question2

$\frac{1}{3 \cdot 5}+\frac{1}{5 \cdot 7}+\frac{1}{7 \cdot 9}+\ldots$ **to 24 terms** $=$

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Options:

A.

$\frac{23}{147}$

B.

$\frac{6}{35}$

C.

$\frac{6}{37}$

D.

$\frac{8}{51}$

Answer: D

Solution:

$$
\begin{aligned}
& S_n=\frac{1}{3 \cdot 5}+\frac{1}{5 \cdot 7}+\frac{1}{7 \cdot 9}+\ldots \\
& S_{24}=\sum_{n=1}^{24} \frac{1}{(2 n+1)(2 n+3)} \\
& =\frac{1}{2} \sum_{n=1}^{24} \frac{1}{2 n+1}-\frac{1}{2 n+3} \\
& =\frac{1}{2}\left[\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{9}\right)+\ldots+\left(\frac{1}{49}-\frac{1}{51}\right)\right] \\
& =\frac{1}{2}\left(\frac{1}{3}-\frac{1}{51}\right)=\frac{1}{2}\left(\frac{16}{51}\right)=\frac{8}{51} .
\end{aligned}
$$

Question3

$1+\frac{4}{15}+\frac{4 \cdot 10}{15 \cdot 30}+\frac{4 \cdot 10 \cdot 16}{15 \cdot 30 \cdot 45}+\ldots . .+\infty=$

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Options:

A.

$\left(\frac{3}{5}\right)^{2/3}$

B.

$\left(\frac{5}{3}\right)^{2/3}$

C.

$\left(\frac{3}{5}\right)^{3/2}$

D.

$\left(\frac{5}{3}\right)^{3/2}$

Answer: B

Solution:

$1+\frac{4}{15}+\frac{4 \cdot 10}{15 \cdot 30}+\frac{4 \cdot 10 \cdot 16}{15 \cdot 30 \cdot 45}+\ldots+\infty$

We know that

$$\begin{aligned}(1+x)^n=1+n x+ & \frac{n(n-1)}{2!} x^2 \\ & +\frac{n(n-1)(n-2)}{3!} x^3+\ldots\end{aligned}$$

On comparing with given expansion, we get

$n x=\frac{4}{15} \Rightarrow x=\frac{4}{15 n} \quad \ldots(i)$

and $\frac{n(n-1)}{2!} x^2=\frac{4 \cdot 10}{15 \cdot 30}=\frac{40}{450}=\frac{4}{45}$

Using Eq. (i), $\frac{n(n-1)}{2!}\left(\frac{4}{15 n}\right)^2=\frac{4}{45}$

$$\begin{aligned}\Rightarrow & \frac{n(n-1)}{2} \times \frac{4 \times 4}{15 n \times 15 n} & =\frac{4}{45} \\ \Rightarrow & \frac{n(n-1)}{2} \times \frac{16}{225 n^2} & =\frac{4}{45} \\ \Rightarrow & & n=\frac{-2}{3}\end{aligned}$$

Also, $x=\frac{4}{15}\left(\frac{-3}{2}\right)=\frac{-2}{5}$

$$\therefore \quad (1+x)^n=\left(1-\frac{2}{5}\right)^{-2/3}$$
$$=\left(\frac{3}{5}\right)^{-2/3}=\left(\frac{5}{3}\right)^{2/3}$$

Question4

If $t_n=\frac{1}{4}(n+2)(n+3), n\in N$, then which one of the following is true?

Assertion (A) $\frac{1}{t_1}+\frac{1}{t_2}+\ldots+\frac{1}{t_{2003}}=\frac{2003}{3009}$

Reason (R) $\frac{1}{t_1}+\frac{1}{t_2}+\ldots+\frac{1}{t_n}=\frac{4n}{(2n+3)}$

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Options:

A.

(A) and (R) are true and (R) is a correct explanation of (A)

B.

(A) and (R) are true, but (R) is not the correct explanation of (A)

C.

(A) is true, (R) is false

D.

(A) is false, (R) is false

Answer: D

Solution:

Given, $t_n=\frac{1}{4}(n+2)(n+3), n\in N \qquad \ldots$ (i)

Also, $\frac{1}{t_1}+\frac{1}{t_2}+\ldots+\frac{1}{t_{2003}}=\frac{2003}{3009}$ $\quad\ldots(ii)$

From Eq. (i), we get

$$\begin{aligned}\frac{1}{t_n}&=\frac{4}{(n+2)(n+3)}\\&=\frac{4}{n+2}-\frac{4}{n+3}\end{aligned}$$

(By partial fraction)

$$\begin{aligned}\Rightarrow\quad&\sum_{n=1}^{2003}\frac{1}{t_n}=\sum_{n=1}^{2003}\left(\frac{4}{n+2}-\frac{4}{n+3}\right)\\&=4\left(\frac{1}{3}-\frac{1}{2006}\right)=4\left(\frac{2006-3}{3\times2006}\right)\\&=4\times\frac{2003}{3\times2006}\\&=\frac{8012}{6018}=\frac{4006}{3009}\end{aligned}$$

So, Assertion is false.

Also, by putting $n=2003$ in Reason then we observe that it is wrong.

Question5

The sum of all integers between 1 and 100 (both inclusive) which are divisible by 5 or 13 is

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Options:

A.

1349

B.

1536

C.

1237

D.

1479

Answer: A

Solution:

The integers divisible by 5 from 1 to 100 are $5, 10, 15, 20, 25, \ldots, 100$. Which are in A.P. with first term (a) $= 5$ and common difference $(d) = 5$) last term $(l) = 100$

So, total number of terms,

$$\begin{aligned} (n) &= \frac{l-a}{d} + 1 = \frac{100-5}{5} + 1 = \frac{95}{5} + 1 \\ &= 19 + 1 = 20 \\ \therefore S_n &= \frac{n}{2}(a+l) \\ &= 10(5+100) = 1050 \end{aligned}$$

Similarly, the integers divisible by 13 from 1 to 100 are $13, 26, 39, 52, \ldots, 91$ which are in AP with to $a = 13, d = 13, l = 91$.

$$So,\ n = \tfrac{l-a}{d} + 1 = \tfrac{91-13}{13} + 1 = \tfrac{78}{13} + 1$$
$$= 6 + 1 = 7$$
$$\therefore \quad S_n = \frac{7}{2}(13+91) = 7 \times 52 = 364$$

Now, the integer divisible by both 5 and 13 from 1 to 100 is 65 .

Hence, the sum of integer from 1 to 100 that are divisible by 5 or 13 is $= 1050 + 364 - 65 = 1414 - 65 = 1349$

Question6

If $x > \sqrt{3}$ and $\frac{x^2+1}{(x^2+2)(x^2+3)}$ is expanded in terms of powers of x, then the coefficient of x^{-8} is

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Options:

A.

0

B.

-81

C.

46

D.

-46

Answer: D

Solution:

Given, $x > \sqrt{3} \Rightarrow x^2 > 3 \Rightarrow 0 < \frac{1}{x^2} < \frac{1}{3}$ Also,

$\frac{x^2+1}{(x^2+2)(x^2+3)} = \frac{-1}{x^2+2} + \frac{2}{x^2+3} = (-1)\left(x^2+2\right)^{-1} + 2\left(x^2+3\right)^{-1}$

$$= (-1)\left(x^2\right)^{-1}\left(1+\frac{2}{x^2}\right)^{-1} + 2\left(x^2\right)^{-1}\left(1+\frac{3}{x^2}\right)^{-1}$$

$$= \frac{-1}{x^2}\left(1-\frac{2}{x^2}+\frac{4}{x^4}-\frac{8}{x^6}+\frac{16}{x^8}-\ldots\infty\right)$$

$$+\frac{2}{x^2}\left(1-\frac{3}{x^2}+\frac{9}{x^4}-\frac{27}{x^6}+\ldots\infty\right)$$

So, the coefficient of

$x^{-8} = (-1)(-8) + (2)(-27)$

$= 8 - 54 = -46$

Question7

$$\sum_{k=1}^{n} k(k+1)(k+2)\ldots(k+r-1) =$$

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Options:

A.

$\frac{n(n+1)(n+2)\ldots(n+r)}{r+1}$

B.

$\frac{n(n+1)(n+2)\ldots(n+r-1)}{r}$

C.

$\frac{n(n+1)(n+2)\ldots(n+r+1)}{r+1}$

D.

$\frac{n(n+1)(n+2)\cdot\cdot 2n}{2n+1}$

Answer: A

Solution:

From the given, we have

$$\sum_{k=1}^{n} k(k+1)(k+2\ldots(k+r-1)$$
$$=\sum_{K=1}^{n}\frac{(k+r-1)!}{(k-1)!}$$
$$=r!\sum_{k=1}^{n}{}^{k+r-1}C_r$$
$$=r!^{n+r}C_{r+1}$$
$$=\frac{n(n+1)(n+2)\ldots(n+r)}{(r+1)}$$

Question8

For all $n\in N$, $\frac{3^n-1}{2}\geq$

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Options:

A.

$n^2\left(2^{\frac{n}{2}}\right)$

B.

$n^2\left(3^{\frac{n-1}{2}}\right)$

C.

$n^3\left(3^{\frac{n-1}{2}}\right)$

D.

$n\left(3^{\frac{n-1}{2}}\right)$

Answer: D

Solution:

$$\text{AM} \geq \text{GM}$$

$$\therefore \quad \frac{1+3+3^2+3^3+3^{n-1}}{n}$$

$$\geq \left(1 \cdot 3 \cdot 3^2 \cdot 3^3 \cdot \ldots \cdot 3^{n-1}\right)^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{3^n-1}{2n} \geq \left(3^{1+2+3+\ldots n-1}\right)^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{3^n-1}{2} \geq n\left(3^{n\frac{(n-1)}{2}}\right)^{\frac{1}{n}}$$

$$\Rightarrow \quad \frac{3^n-1}{2} \geq n3^{\frac{n-1}{2}}$$

Question9

If $2 \cdot 5+5 \cdot 9+8 \cdot 13+11 \cdot 17+\ldots$ **to** n **terms** $=an^3+bn^2+cn+d$ **, then** $a-b+c-d=$

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Options:

A.

7

B.

5

C.

-3

D.

-1

Answer: D

Solution:

Given, $2 \cdot 5 + 5 \cdot 9 + 8 \cdot 13 + 11 \cdot 17 + \ldots +$ upto n terms

$t_n = (3n-1)(4n+1)$

$t_n = 12n^2 - n - 1$

$\therefore S_n = \Sigma t_n = \Sigma\left(12n^2 - n - 1\right) = 12\Sigma n^2 - \Sigma n - \Sigma 1$

$= 12 \cdot \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - n$

$= n\left[2\left(2n^2 + 3n + 1\right) - \frac{n+1}{2} - 1\right] = \frac{n}{2}\left[8n^2 + 12n + 4 - n - 1 - 2\right]$

$= \frac{n}{2}\left[8n^2 + 11n + 1\right]$

$= 4n^3 + \frac{11}{2}n^2 + \frac{n}{2} = an^3 + bn^2 + cn + d$

$\Rightarrow a = 4, b = \frac{11}{2}, c = \frac{1}{2}$ and $d = 0$

$\therefore a - b + c - d = 4 - \frac{11}{2} + \frac{1}{2} - 0 = 4 - 5 = -1$

Question10

For all $n \in N$, if $1^3 + 2^3 + 3^3 + \ldots n^3 > x$, then a value of x among the following is

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Options:

A.

$\frac{n^2}{4}$

B.

n^2

C.

n^4

D.

$\frac{n^2(n+1)^2}{4}$

Answer: A

Solution:

We have, $1^3+2^3+3^3+\ldots+n^3>x$

$$\because \quad \Sigma n^3=\left[\frac{n(n+1)}{2}\right]^2=\frac{n^2}{4}(n+1)^2>x$$

$$\because \quad \frac{n^2}{4}<\frac{n^2}{4}(n+1)^2$$

$$\therefore \quad x=\frac{n^2}{4}$$

Question11

The nth term of the series $1+(3+5+7)+(9+11+13+15+17)+\ldots$ is

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Options:

A. $(2n+1)\left[n^2-(n-1)^2\right]$

B. $(2n-1)\left[(n-1)^2-n^2\right]$

C. $(2n+1)\left[(n-1)^2-n^2\right]$

D. $(2n-1)\left[(n-1)^2+n^2\right]$

Answer: D

Solution:

We have,

$1+(3+5+7)+(9+11+13+15+17)+\ldots$

Total number of terms

$$\begin{aligned} & =1+3+5+7+\ldots+n \text{ terms} \\ & =\frac{n}{2}[2+(n-1)2] \\ & =n^2 \end{aligned}$$

$S_n=1+3+5+7+9+11+13+15+17+\ldots$

upto n^2 terms

$$
\begin{aligned}
S_n &= \frac{n^2}{2}\left[2+\left(n^2-1\right)2\right] \\
&= \frac{n^2}{2} \times \left(2n^2\right) = n^4 \\
a_n &= S_n - S_{n-1} \\
&= n^4 - (n-1)^4 \\
&= (n-(n-1))(n+(n-1))\left(n^2+(n-1)^2\right) \\
&= (2n-1)\left[n^2+(n-1)^2\right]
\end{aligned}
$$

Question12

The number of ways of selecting- 3 numbers that are in GP from the set $\{1, 2, 3, 100\}$ is

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Options:

A. 18

B. 52

C. 14

D. 53

Answer: D

Solution:

Here, a i.e. first term ≥ 1

ar^2(third term) ≤ 100

$$
\begin{aligned}
& r^2 \leq 100 \\
\Rightarrow \quad & r \leq 10
\end{aligned}
$$

When $r = 2$, then $a \leq \frac{100}{4} = 25$

$\therefore a$ can have 25 values

When $r = 3$, then $a \leq \frac{100}{9} \Rightarrow a \leq 11$

$\therefore a$ can have 11 values

When $r = 4$, then $a \leq \frac{100}{16} \Rightarrow a \leq 6$

$\therefore a$ can have 6 values.

When $r=5$, then $a \leq \frac{100}{25}=4$

$\therefore a$ can have 4 values.

When $r=6$, then $a \leq \frac{100}{36} \Rightarrow a \leq 2$

$\therefore a$ can have 2 values.

When $r=7$, then $a \leq \frac{100}{49} \Rightarrow a \leq 2$

$\therefore a$ can have 2 values.

When $r=8,9$ and 10 , then

$a \leq \frac{100}{64}, a \leq \frac{100}{81}$ and $a \leq \frac{100}{100}$,

respectively.

$\therefore a$ can have 1 value in each case. Hence, total number of ways.

$$
\begin{aligned}
& =25+11+6+4+2+2+1+1+1 \\
& =53
\end{aligned}
$$

Question13

$2+3+5+6+8+9+\ldots .2 n$ terms $=$

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Options:

A. $3 n^2+2 n$

B. $4 n^2+2 n$

C. $4 n^2$

D. $5 n^2+2 n$

Answer: A

Solution:

$2+3+5+6+8+9 \ldots 2 n$ terms

Observe the sequence and note that every third term is missing.

So, we can split this summation into two Arithmatic Progiression (AP's).

First $AP_1 = 2 + 5 + 8 + \ldots n$ terms

where, $a_1 = 2, d_1 = 3$

Then, $S_1 = \frac{n}{2}[2a_1 + (n-1)d_1]$

Similarly, $AP_2 = 3 + 6 + 9 + \ldots n$ terms

where, $a_2 = 3, d_2 = 3$

Then, $S_2 = \frac{n}{2}[2a_2 + (n-1)d_2]$

Therefore, $S = S_1 + S_2$

$$\begin{aligned} S =& \frac{n}{2}[2 \times 2 + (n-1)3] + \frac{n}{2}[2 \times 3 + (n-1) \times 3] \\ =& \frac{n}{2}[4 + 3(n-1) + 6 + 3(n-1)] \\ =& \frac{n}{2}[10 + 3n - 3 + 3n - 3] \\ =& \frac{n}{2}[4 + 6n] = \frac{n}{2}[2 + 3n] \times 2 \\ =& 2n + 3n^2 \\ S =& 3n^2 + 2n \end{aligned}$$

Question14

If α, β are the roots of the equation $x^2 - 6x - 2 = 0, \alpha > \beta$ and $a_n = \alpha^n - \beta^n, n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to

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Options:

A. 6

B. 4

C. 3

D. 2

Answer: C

Solution:

When solving the quadratic equation $x^2 - 6x - 2 = 0$, the roots α and β can be found using the quadratic formula:

$\alpha, \beta = \frac{6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{6 \pm \sqrt{36 + 8}}{2} = \frac{6 \pm \sqrt{44}}{2} = 3 \pm \sqrt{11}$

Given that $a_n = \alpha^n - \beta^n$ for $n \geq 1$, and knowing α and β are roots, they satisfy:

$t^2 - 6t - 2 = 0 \implies t^2 = 6t + 2$

Multiplying by t^{n-2}, we have:

$t^n = 6t^{n-1} + 2t^{n-2}$

Therefore, for α:

$\alpha^n = 6\alpha^{n-1} + 2\alpha^{n-2}$

And similarly for β:

$\beta^n = 6\beta^{n-1} + 2\beta^{n-2}$

This gives us for a_n:

$$\begin{aligned} a_n &= \alpha^n - \beta^n \\ &= 6(\alpha^{n-1} - \beta^{n-1}) + 2(\alpha^{n-2} - \beta^{n-2}) \\ a_n &= 6a_{n-1} + 2a_{n-2} \end{aligned}$$

To find $\frac{a_{10} - 2a_8}{2a_9}$, we apply the recurrence relation:

Given $a_{10} = 6a_9 + 2a_8$, we have:

$a_{10} - 2a_8 = 6a_9 + 2a_8 - 2a_8 = 6a_9$

Thus,

$\frac{a_{10} - 2a_8}{2a_9} = \frac{6a_9}{2a_9} = 3$

Therefore, the value is 3.

Question15

$|x| < 1$, The coefficient of x^2 in the power series expansion of $\frac{x^4}{(x+1)(x-2)}$ is

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Options:

A. 3

B. 0

C. -1

D. -3

Answer: B

Solution:

We have,

$$\frac{x^4}{(x-2)(x+1)}=x^2+x+3+\frac{-\frac{1}{3}}{x+1}+\frac{\frac{16}{3}}{x-2}$$

$$=x^2+x+3+\left(-\frac{1}{3}\right)(1+x)^{-1}+\frac{16}{3}(x-2)^{-1}$$

$$=x^2+x+3-\tfrac{1}{3}\left[1-x+x^2-x^3+\ldots\right]$$
$$-\tfrac{8}{3}\left(1-\tfrac{x}{2}\right)^{1}$$

$$=x^2+x+3-\frac{1}{3}\left[1-x+x^2-x^3+\ldots\right]$$
$$-\frac{8}{3}\left[1+\frac{x}{2}+\frac{x^2}{4}+\ldots\right]$$

Hence, coefficient of x^2 is $\left(1-\frac{1}{3}-\frac{2}{3}\right)$

$=(1-1)=0$

Question16

If $1\cdot3\cdot5+3\cdot5\cdot7+5\cdot7\cdot9+\ldots n$ terms $=n(n+1)f(n)-3n$, then $f(l)=$

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Options:

A. 9

B. 11

C. 12

D. 8

Answer: A

Solution:

To solve the given problem, we need to analyze the expression for the sum of terms and match it with the provided equation.

The sequence is structured as:

$1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 + 5 \cdot 7 \cdot 9 + \ldots$

We define the r-th term as:

$a_r = (2r-1)(2r+1)(2r+3)$

Calculating a_r, we expand:

$a_r = (2r-1)(2r+1)(2r+3) = 8r^3 + 12r^2 - 2r - 3$

The sum S_n of the first n terms is:

$S_n = \sum_{r=1}^{n} a_r = \sum_{r=1}^{n} \left(8r^3 + 12r^2 - 2r - 3\right)$

Breaking it down, we compute:

$S_n = 8\sum_{r=1}^{n} r^3 + 12\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$

Substituting the formula for each sum, we have:

$8\left[\frac{n(n+1)}{2}\right]^2 + 12\left[\frac{n(n+1)(2n+1)}{6}\right] - 2\left[\frac{n(n+1)}{2}\right] - 3n$

Simplifying, this becomes:

$S_n = n(n+1)(2n^2 + 6n + 1) - 3n$

Here, the function $f(n)$ is:

$f(n) = 2n^2 + 6n + 1$

To find $f(1)$, substitute $n = 1$:

$f(1) = 2(1)^2 + 6(1) + 1 = 9$

Thus, the function $f(l)$ evaluates to 9.

Question17

The condition that the roots of $x^3 - bx^2 + cx - d = 0$ are in arithmetic progression is

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Options:

A. $9cb = 2b^3 + 27d$

B. $9cb = 2d^3 + 27b$

C. $9cb = 2d^3 + 27b$

D. $9cd = 2b^3 + 27d$

Answer: A

Solution:

Given the polynomial equation $x^3 - bx^2 + cx - d = 0$, we need to find the condition for its roots to be in arithmetic progression.

Let the roots of this polynomial be α, β, and γ. Since these roots are in arithmetic progression, they satisfy the condition:

$2\beta = \alpha + \gamma$

We also know from the properties of polynomials that the sum of the roots is equal to the coefficient of the x^2 term, which is b. Thus,

$\alpha + \beta + \gamma = b$

Substituting the condition of arithmetic progression:

$3\beta = b \quad \Rightarrow \quad \beta = \frac{b}{3}$

Next, we use the condition for the sum of the products of the roots taken two at a time:

$\alpha\beta + \beta\gamma + \gamma\alpha = c$

And for the product of the roots:

$\alpha\beta\gamma = d$

Substituting $\beta = \frac{b}{3}$, the product becomes:

$\alpha\gamma \cdot \frac{b}{3} = d \quad \Rightarrow \quad \alpha\gamma = \frac{3d}{b}$

Now substituting the values of β and $\alpha\gamma$ into the equation for the sum of the products of the roots taken two at a time:

$\alpha \cdot \frac{b}{3} + \frac{b}{3}\gamma + \frac{3d}{b} = c$

Simplifying further:

$\frac{b}{3}(\alpha + \gamma) + \frac{3d}{b} = c$

Using the condition $\alpha + \gamma = 2\beta$:

$\frac{b}{3} \cdot 2\beta + \frac{3d}{b} = c$

Substituting $\beta = \frac{b}{3}$:

$\frac{b}{3} \cdot \frac{2b}{3} + \frac{3d}{b} = c$

Simplifying this expression leads to:

$\frac{2b^2}{9} + \frac{3d}{b} = c$

By multiplying through by $9b$ to clear denominators, we become:

$2b^3 + 27d = 9bc$

Thus, the condition for the roots of the polynomial to be in arithmetic progression is:

$9bc = 2b^3 + 27d$

Question18

In the expansion of $\frac{2x+1}{(1+x)(1-2x)}$ the sum of the coefficients of the first 5 odd powers of x is

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Options:

A. $\frac{5}{3} + \frac{8}{9}\left(4^5 - 1\right)$

B. $\frac{5}{3} + \frac{8}{3}\left(4^5 - 1\right)$

C. $-\frac{5}{3} + \frac{8}{9}\left(4^5 - 1\right)$

D. $\frac{5}{3} + \frac{8}{12}\left(4^5 + 1\right)$

Answer: A

Solution:

Given expression $\frac{2x+1}{(1+x)(1-2x)}$

By partial fraction

$$\frac{(2x+1)}{(1+x)(1-2x)} = \frac{A}{(1+x)} + \frac{B}{(1-2x)} \qquad \ldots(i)$$
$$2x + 1 = A(1-2x) + B(1+x)$$
$$2x + 1 = A + B + (-2A + B)x$$

On comparing coefficients both sides, we get

$$A + B = 1 \Rightarrow A = 1 - B \qquad \ldots(ii)$$
$$-2A + B = 2 \qquad \ldots(iii)$$

From Eqs. (ii) and (iii), we get

$$-2 + 2B + B = 2$$
$$\Rightarrow \qquad B = \frac{4}{3} \text{ and } A = -\frac{1}{3}$$

On putting the value of A and B in Eq.

(i), we get

$$\frac{2x+1}{(1+x)(1-2x)} = -\frac{1}{3(1+x)} + \frac{4}{3(1-2x)}$$
$$= -\frac{1}{3}(1+x)^{-1} + \frac{4}{3}(1-2x)^{-1}$$

$$= -\frac{1}{3}\left[1 - x + x^2 - x^3 + x^4 \quad -x^5 + x^6 - x^7 + x^8 - x^9\right] + \frac{4}{3}\left[1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5\right]$$
$$64x^6 + 128x^7 + 256x^8 + 512x^9 + \ldots]$$

$$= \left(-\frac{1}{3} + \frac{4}{3}\right) + \left(\frac{1}{3} + \frac{1}{8}\right)x + \left(-\frac{1}{3} + \frac{16}{3}\right)x^2 + \left(\frac{1}{3} + \frac{32}{3}\right)x^3 + \left(-\frac{1}{3} + \frac{64}{3}\right)x^4 + \left(\frac{1}{3} + \frac{128}{3}\right)x^5$$
$$+ \left(-\frac{1}{3} + \frac{256}{3}\right)x^6 + \left(\frac{1}{3} + \frac{512}{3}\right)x^7 \quad + \left(-\frac{1}{3} + \frac{1024}{3}\right)x^8 + \left(\frac{1}{3} + \frac{2024}{3}\right)x^5$$

Now, sum of coefficients of first 5 odd power of x, we get

$$= \left(\frac{1}{3} + \frac{8}{3}\right) + \left(\frac{1}{3} + \frac{32}{3}\right) + \left(\frac{1}{3} + \frac{128}{3}\right) + \left(\frac{1}{3} + \frac{512}{3}\right) + \left(\frac{1}{3} + \frac{2048}{3}\right)$$
$$= \left[\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right] \quad + \left[\frac{8}{3} + \frac{32}{3} + \frac{128}{3} + \frac{512}{3} + \frac{2048}{3}\right]$$

$$= \frac{5}{3} + \frac{1}{3}[2728]$$
$$= \frac{5}{3} + \frac{8}{9}(1023)$$
$$= \frac{5}{3} + \frac{8}{9}\left(4^5 - 1\right)$$

Question19

$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \frac{1}{9 \cdot 13} + \ldots$ **upto** n **terms** $=$

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Options:

A. $\frac{1}{4n+1}$

B. $\frac{4}{4n+1}$

C. $\frac{n}{4n+1}$

D. $\frac{4n+1}{5(4n+1)}$

Answer: C

Solution:

To understand the summation of the given series, consider the series:

$\frac{1}{1\cdot5}+\frac{1}{5\cdot9}+\frac{1}{9\cdot13}+\ldots$ up to n terms

We denote this sum as S:

$S=\frac{1}{1\cdot5}+\frac{1}{5\cdot9}+\frac{1}{9\cdot13}+\ldots+\frac{1}{(4n-3)(4n+1)}$

We can express each term in a telescoping form:

$S=\frac{1}{4}\left\{\frac{5-1}{1\cdot5}+\frac{9-5}{5\cdot9}+\frac{13-9}{9\cdot13}+\ldots+\frac{(4n+1)-(4n-3)}{(4n-3)(4n+1)}\right\}$

Simplifying further using partial fractions, we have:

$=\frac{1}{4}\left[\left(1-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{13}\right)+\ldots+\left(\frac{1}{4n-3}-\frac{1}{4n+1}\right)\right]$

Observe that it forms a telescoping series, where most middle terms cancel out. Thus, we are left with:

$=\frac{1}{4}\left[1-\frac{1}{(4n+1)}\right]$

Finally, we express this as:

$=\frac{1}{4}\left[\frac{4n+1-1}{4n+1}\right]=\frac{1}{4}\cdot\frac{4n}{4n+1}=\frac{n}{4n+1}$

Thus, the sum of the series up to n terms is:

$\frac{n}{4n+1}$

Question20

If the roots of the equation $4x^3-12x^2+11x+m=0$ are in arithmetic progression, then $m=$

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Options:

A. -3

B. 1

C. 2

D. 3

Answer: A

Solution:

To solve for m in the equation $4x^3 - 12x^2 + 11x + m = 0$, given that the roots are in arithmetic progression, we proceed as follows:

Assume the roots of the polynomial are $a - d$, a, and $a + d$.

Sum of Roots:

According to the polynomial equation, the sum of the roots is given by:

$(a - d) + a + (a + d) = \frac{-b}{a} = \frac{12}{4} = 3$

Simplifying the left-hand side:

$3a = 3$

Solving for a, we find:

$a = 1$

Using the Root in the Original Equation:

Since $a = 1$ is a root of the polynomial, it should satisfy the equation:

$4(1)^3 - 12(1)^2 + 11(1) + m = 0$

Simplify this:

$4 - 12 + 11 + m = 0$

$3 + m = 0$

Solving for m:

$m = -3$

Thus, the value of m is -3.

Question21

If $2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by k for all $n \in N$, then $k =$

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Options:

A. 209

B. 11

C. 8

D. 3

Answer: B

Solution:

Given, $P(n) = 2 \cdot 4^{2n+1} + 3^{3n+1}$

For $n = 1$

$$P(1) = 2 \cdot 4^{2+1} + 3^{3+1} = 2 \cdot 4^3 + 3^4$$
$$= 2 \cdot 64 + 81 = 128 + 81 = 209$$

For $n = 2$

$$P(2) = 2 \cdot 4^{4+1} + 3^{6+1} = 2 \cdot 4^5 + 3^7$$
$$= 2048 + 2187 = 4235$$

Now, HCF of 209 and 4235 is 11.

$\because 2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by 11 ,

$\forall n \in N$.

$\therefore \quad k = 11$

Question22

If the roots of the equation $x^3 + ax^2 + bx + c = 0$ are in arithmetic progression. Then,

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Options:

A. $a^3 - 3ab + c = 0$

B. $9ab = 2a^3 + 27c$

C. $a^2 - 2bc + c = 0$

D. $3ab - 3c - a^3 = 0$

Answer: B

Solution:

To solve the problem where the roots of the polynomial equation $x^3 + ax^2 + bx + c = 0$ are in arithmetic progression, let's denote these roots as $s - t$, s, and $s + t$.

Sum of Roots: From Vieta's formulas, we know the sum of the roots is equal to $-a$. Therefore,

$$(s - t) + s + (s + t) = -a$$

Simplifying gives:

$3s = -a \quad \Rightarrow \quad a = -3s$

Sum of Products of Roots Taken Two at a Time: The sum of the products of the roots taken two at a time is equal to b. Thus,

$(s-t)s + s(s+t) + (s-t)(s+t) = b$

This expands to:

$s^2 - st + s^2 + st + s^2 - t^2 = b$

Simplifying gives:

$3s^2 - t^2 = b$

Product of Roots: The product of the roots is equal to $-c$. Therefore,

$(s-t)s(s+t) = -c$

This simplifies to:

$s^3 - st^2 = -c$

So, we have:

$c = s^3 - st^2$

Calculating $2a^3 + 27c$ and $9ab$:

$2a^3 + 27c = 2(-3s)^3 + 27(-s^3 + st^2)$

Calculating gives:

$= 2(-27s^3) + 27(-s^3 + st^2)$

$= -54s^3 - 27s^3 + 27st^2$

$= -81s^3 + 27st^2$

For $9ab$:

$9ab = 9(-3s)(3s^2 - t^2)$

$= -81s^3 + 27st^2$

Therefore,

$9ab = 2a^3 + 27c$

The equation $9ab = 2a^3 + 27c$ holds true when the roots of the polynomial are in arithmetic progression.

Question23

$\frac{1}{3\cdot 7} + \frac{1}{7\cdot 11} + \frac{1}{11\cdot 15} + \ldots$ **to 50 terms** $=$

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Options:

A. $\frac{50}{203}$

B. $\frac{50}{609}$

C. $\frac{150}{203}$

D. $\frac{25}{609}$

Answer: B

Solution:

$\frac{1}{3\cdot 7}+\frac{1}{7\cdot 11}+\frac{1}{11\cdot 15}+\ldots$ to 50 terms

We can see that each term in the series can be represented as $\frac{1}{(4n-1)(4n+3)}$

So, $S=\sum_{n=1}^{50}\frac{1}{(4n-1)(4n+3)}$

Using partial fraction

$S=\sum_{n=1}^{50}\frac{1}{4}\left[\frac{1}{4n-1}-\frac{1}{4n+3}\right]$

$$=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}+\frac{1}{11}-\frac{1}{15}\cdots\frac{1}{199}-\frac{1}{203}\right]$$
$$=\frac{1}{4}\left[\frac{1}{3}-\frac{1}{203}\right]=\frac{1}{4}\left[\frac{203-3}{609}\right]=\frac{1}{4}\left[\frac{200}{609}\right]=\frac{50}{609}$$

Question24

$$1+\frac{1}{3}+\frac{1\cdot 3}{3\cdot 6}+\frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9}+\ldots \text{ to } \infty=$$

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Options:

A. $\sqrt{5}$

B. $\sqrt{6}$

C. $\sqrt{15}$

D. $\sqrt{3}$

Answer: D

Solution:

We are given the infinite series:

$1 + \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \ldots$

This series can be related to the binomial expansion of $(1+x)^n$, which is given by:

$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \ldots$

Comparing this with the terms of our series, we identify:

$nx = \frac{1}{3}$ implies $x = \frac{1}{3n}$.

For the second comparison, $\frac{n(n-1)x^2}{2} = \frac{1}{6}$.

Substitute $x = \frac{1}{3n}$ into the second equation:

$\frac{n(n-1)}{2} \times \left(\frac{1}{3n}\right)^2 = \frac{1}{6}$

Simplifying gives:

$\frac{n(n-1)}{18n^2} = \frac{1}{6}$

$\frac{n-1}{3n} = 1$

This simplifies to:

$3n - n = -1$

$2n = -1$

$n = -\frac{1}{2}$

Substituting back to find x:

$x = \frac{1}{3} \times \frac{-2}{1} = \frac{-2}{3}$

Thus, the series becomes:

$(1+x)^n = \left(1 - \frac{2}{3}\right)^{-1/2} = \left(\frac{1}{3}\right)^{-1/2} = 3^{1/2} = \sqrt{3}$

Therefore, the sum of the series is $\sqrt{3}$.

Question25

$2 \cdot 5 + 5 \cdot 9 + 8 \cdot 13 + 11 \cdot 17 + \ldots$ to 10 terms $=$

Options:

A. 3355

B. 4555

C. 1375

D. 1380

Answer: B

Solution:

We have,

$2 \cdot 5+5 \cdot 9+8 \cdot 13+11 \cdot 17+\ldots+$ upto 10

terms.

$=(3-1)(4+1)+(6-1)(8+1)+(9-1)$
$(12+1)+(12-1)(16+1)+\ldots+$ upto 10

terms.

$$\begin{aligned} & =\sum_{r=1}^{10}(3 r-1)(4 r+1) \\ & =\sum_{r=1}^{10}\left(12 r^2-r-1\right) \\ & =12 \times \frac{10 \times 11 \times 21}{6}-\frac{10 \times 11}{2}-10 \\ & =4620-55-10=4555 \end{aligned}$$

Question26

If the roots of equation $x^3-13 x^2+K x-27=0$ are in geometric progression, then $K=$

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Options:

A. -30

B. 30

C. 39

D. -39

Answer: C

Solution:

We have, the roots of $x^3 - 13x^2 + kx - 27 = 0$ are in GP be.

Let the roots of equation be $\frac{a}{r}, a, ar$

$$\frac{a}{r} \times a \times ar = 27 \Rightarrow a^3 = 27 \Rightarrow a = 3$$
$$\Rightarrow \quad \frac{a}{r} + a + ar = 13$$
$$\Rightarrow \quad \frac{3}{r} + 3 + 3r = 13$$
$$\Rightarrow \quad 3\left(r + \frac{1}{r}\right) = 10$$
$$\Rightarrow \quad 3r^2 - 10r + 3 = 0$$
$$\Rightarrow \quad 3r^2 - 9r - r + 3 = 0$$
$$\Rightarrow \quad (r-3)(3r-1) = 0$$
$$\Rightarrow \quad r = 3, \frac{1}{3}$$

$\therefore$ The roots of the equation are 1,3 and 9 .

$$K = 1 \times 3 + 3 \times 9 + 9 \times 1 = 3 + 27 + 9$$
$$= 39$$

Question27

$$1 - \frac{2}{3} + \frac{2 \cdot 4}{3 \cdot 6} - \frac{2 \cdot 4 \cdot 6}{3 \cdot 6 \cdot 9} + \ldots \infty =$$

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Options:

A. $\frac{3}{5}$

B. $\left(\frac{2}{5}\right)^{\frac{2}{3}}$

C. $\frac{2}{5}$

D. $\left(\frac{3}{5}\right)^{\frac{2}{3}}$

Answer: A

Solution:

Let $S_\infty = 1 - \frac{2}{3} + \frac{2\cdot4}{3\cdot6} - \frac{2\cdot4\cdot6}{3\cdot6\cdot9} + \ldots\infty$
$= \left(1-\frac{2}{3}\right) + \frac{2\cdot4}{3\cdot6}\left(1-\frac{6}{9}\right) + \frac{2\cdot4\cdot6\cdot8}{3\cdot6\cdot9\cdot12}$
$\left(1-\frac{10}{15}\right) + \ldots\infty$

$$= \left(1-\frac{2}{3}\right)\left[1 + \frac{2\cdot4}{3\cdot6} + \frac{2\cdot4\cdot6\cdot8}{3\cdot6\cdot9\cdot12} + \ldots\infty\right]$$

$$= \frac{1}{3}\left[1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + \ldots\infty\right]$$

$$= \frac{1}{3}\left[\frac{1}{1-\left(\frac{2}{3}\right)^2}\right] = \frac{1}{3} \times \frac{9}{5} = \frac{3}{5}$$

Question28

Suppose that the three points A, B and C in the plane are such that their x-coordinates as well as y-coordinates are in GP with the same common ratio. Then, the points A, B and C

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Options:

A. constitute a right angled triangle

B. form an isosceles triangle

C. lie on a straight line

D. form an equilateral triangle

Answer: C

Solution:

Let the coordinate $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$.

According to question, x - coordinate as well as y coordinates are in GP.

Let $x_1 = a, x_2 = ar$ and $x_3 = ar^2, y_1 = b, y_2 = br$ and $y_3 = br^2$

Now, $A(a, b), B(ar, br), C\left(ar^2, br^2\right)$.

Slope of $AB = \frac{b(1-r)}{a(1-r)} = \frac{b}{a}$ and

Slope of BC $= \frac{br(1-r)}{\mathrm{ar}(1-r)} = \frac{b}{a}$

As slope of $AB =$ Slope of BC

$\therefore$ A, B and C are collinear.

$\therefore$ A, B and C lie on a straight line.

Question29

Using mathematical induction, the numbers a_n' s are defined by $a_0 = 1, a_{n+1} = 3n^2 + n + a_n (n \geq 0)$, then a_n is equal to

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Options:

A. $n^3 + n^2 + 1$

B. $n^3 - n^2 + 1$

C. $n^3 - n^2$

D. $n^3 + n^2$

Answer: B

Solution:

Given, $a_0 = 1, a_{n+1} = 3n^2 + n + a_n$ .... (i)

From Eq.(i), we get

$$\begin{aligned} & a_1 = 3(0)^2 + 0 + a_0 = 1 \\ \Rightarrow \quad & a_1 = 1 \\ & a_2 = 3(1)^2 + 1 + a_1 = 3 + 1 + 1 = 5 \\ \Rightarrow \quad & a_2 = 5 \end{aligned}$$

$a_3 = 3(2)^2 + 2 + a_2 = 12 + 2 + 5 = 19$

$\Rightarrow \quad a_3 = 19$

Now, check the option for $n = 3$ which expression satisfy by $a_3 = 19$

Option (a), $a_n = n^3 + n^2 + 1$

Put $n = 3$, then

$$\begin{aligned} a_3 &= (3)^3 + (3)^2 + 1 \\ &= 27 + 9 + 1 = 37 \\ \because \quad a_3 &\neq 37 \\ \therefore \quad a_n &\neq n^3 + n^2 + 1 \end{aligned}$$

Option (b), $a_n = n^3 - n^2 + 1$

Option (b), $a_n = n^3 - n^2 + 1$

Put $n = 3$, then

$$\begin{aligned} a_3 &= (3)^3 - (3)^2 + 1 \\ &= 27 - 9 + 1 = 19 \\ \therefore \quad a_n &= n^3 - n^2 + 1 \end{aligned}$$

Question30

If $1 + x^2 = \sqrt{3}x$, then $\sum_{n=1}^{24} \left(x^n - \frac{1}{x^n}\right)^2$ is equal to

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Options:

A. 48

B. -48

C. -24

D. 24

Answer: B

Solution:

Given, $x^2-\sqrt{3}x+1=0$

$$\Rightarrow \quad x=\frac{\sqrt{3}\pm\sqrt{3-4}}{2}$$

$$x=\frac{\sqrt{3}}{2}\pm\frac{1}{2}i=\cos\frac{\pi}{6}\pm i\sin\frac{\pi}{6}$$

$$\Rightarrow \quad x^{2n}=\cos\frac{2n\pi}{6}\pm i\sin\frac{2n\pi}{6}$$

$$\Rightarrow \quad x^{2n}=\cos\frac{n\pi}{3}\pm i\sin\frac{n\pi}{3}$$

and $x^{-2n}=\cos\frac{n\pi}{3}\pm i\sin\frac{n\pi}{3}$

Now, $\left(x^n-\frac{1}{x^n}\right)^2=\left(x^{2n}+\frac{1}{x^{2n}}-2\right)$

$$=-2+2\cos\frac{n\pi}{3}$$

$$\Rightarrow \sum_{n=1}^{24}\left(x^n-\frac{1}{x^n}\right)^2=\sum_{n=1}^{24}\left(-2+2\cos\frac{n\pi}{3}\right)$$

$$=-48+2\left[\cos\frac{\pi}{3}+\cos\frac{2\pi}{3}+\ldots+\cos\frac{24\pi}{3}\right]$$

$$=-48+2\left[\frac{\cos\left(\frac{2\pi}{3}+\frac{23\pi}{6}\right)\cdot\sin\frac{24\pi}{6}}{\sin\frac{\pi}{6}}\right]$$

$$=-48+2\cdot 0=-48$$

Question31

Let p and q be the roots of the equation $x^2-2x+A=0$ and let r and s be the roots of the equation $x^2-18x+B=0$. If $p<q<r<s$ are in AP then the values of A and B are

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Options:

A. $-3, 77$

B. $3, -77$

C. $3, 77$

D. $-3, -77$

Answer: A

Solution:

$\because p, q, r, s$ are in AP.

Let $p = a, q = a + d, r = a + 2d, s = a + 3d$

Given, p, q are roots of

$x^2 - 2x + A = 0$

Then, $p + q = 2$ and $pq = A$

$\Rightarrow 2a + d = 2$ .... (i)

Also, r and s are roots of $x^2 - 18x + B = 0$

Then, $r + s = 18$ and $rs = B$

$\Rightarrow 2a + 5d = 18$ ..... (ii)

Solving Eqs. (i) and (ii), we get

$a = -1, d = 4$

Now, $pq = A$ (product of roots)

or $A = a(a + d) = (-1)(-1 + 4)$

$A = -3$

and $rs = B$

or $B = (a + 2d)(a + 3d)$

$= (-1 + 8)(-1 + 12) = 77$

$\therefore A = -3, B = 77$

Question32

Let $f(x) = x^3 + ax^2 + bx + c$ be polynomial with integer coefficients. If the roots of $f(x)$ are integer and are in Arithmetic Progression, then a cannot take the value

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Options:

A. -642

B. 1214

C. 1323

D. 1626

Answer: B

Solution:

$f(x)=x^3+a x^2+b x+c$

Let roots of $f(x)$ are α, β, γ and since roots are in AP.

$\therefore \quad 2 \beta=\alpha+\gamma$

Now, Sum of roots

$\alpha+\beta+\gamma=-a$

or $2 \beta+\beta=-a$ or $\beta=-\frac{a}{3}$

It is given that, roots are integers.

$\therefore \beta$ is an integer when a is multiple of 3 .

Option (a), (c) and (d) are multiple of 3.

$\Rightarrow \quad a \neq 1214$

Option (b) is correct.
